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VAR LIMITS FOR PENSION FUNDS: AN
EVALUATION

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VaR Limits for Pension Funds: An Evaluation*

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Abstract

This paper evaluates the effects of VaR limits and quantitative restrictions on portfolio choices in the context of a risk-based supervision framework. It shows the conditions under which Value-at-Risk (VaR) constraints are equivalent to constraints on volatility. The paper also presents some considerations that regulators should take into account when adopting a risk-based supervision framework.

Key Words: Portfolio Choice, VaR.

JEL Classification: G11, G32.

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1 Introduction

A risk-based approach for supervision and regulation of the financial sector is gaining ground. Market risk is one of the components that regulators attempt to measure, monitor, and mitigate. The VaR measure has been one of the possibilities that have been explored for this purpose. One of the most important sectors in which this practice has been adopted is the pension funds industry. For example, Mexico has adopted a regulation that combines quantitative limits and a Value-at-Risk (VaR) limit and some other countries are considering adopting a similar framework.

This paper evaluates the effects of VaR limits and quantitative restrictions on portfolio choices. The paper is organized as follows: Section 2 presents some equivalences between VaR limits and conventional risk measures. These equivalences are used to obtain some implications with respect to the effects of VaR limits. Section 3 uses Chilean data to exemplify these effects and discuss specific characteristics of VaR based supervision.¹ Finally, Section 4 presents some recommendations.

2 Some equivalences

This section derives some equivalences among commonly used strategies for portfolio selection. These strategies share the property of choosing portfolios that combine returns and volatility such that the investor's objective function is maximized. For instance, the mean-variance frontier approach implies that the portfolio is chosen such that volatility is minimized subject to the constraint of obtaining a certain expected return. It is shown that this strategy is equivalent to one that assumes quadratic preferences and therefore maximizes a utility function that is increasing in expected return and decreasing in volatility. Finally, under certain circumstances, these strategies are also equivalent to a VaR approach, under which the investment manager chooses a portfolio that maximizes expected returns subject to the constraint that the probability of a loss beyond a given amount is set at a fixed level.

The circumstance under which this last result holds is when the distribution of returns is elliptical, which generally implies symmetry. Thus, this result is not restricted to the case of normality. The equivalences that are derived are useful to discern the likely effects of imposing VaR limits.

2.1 Quadratic preferences and the mean-variance frontier

Let there be n risky assets with mean vector m and covariance matrix V . Define w_a as the n -vector of portfolio weights for an arbitrary portfolio a with weights summing to unity. The mean return and variance of this portfolio are denoted by $\mu_a = w_a' m$ and $\sigma_a^2 = w_a' V w_a$ respectively.

¹We use Chilean data because of its availability. The results obtained are not country specific.

Definition 1 *Portfolio \mathbf{p} is the minimum-variance portfolio of all portfolios with mean return μ_p if its portfolio weight vector is the solution to the constrained optimization problem:*

$$\min_w \frac{1}{2} w' V w \quad (1)$$

subject to

$$w' \mathbf{1} = 1 \quad (2)$$

$$w' m = \mu_p. \quad (3)$$

The first order conditions for this problem are:

$$V w_p - \lambda_1 \mathbf{1} - \lambda_2 m = 0. \quad (4)$$

where $\mathbf{1}$ is an n -vector of ones, and λ_1 and λ_2 are the Lagrange multipliers of (2) and (3) respectively.

Combining (2), (3), and (4), the solution:

$$w_p = G + H \mu_p \quad (5)$$

is obtained, where G and H are n -vectors,

$$\begin{aligned} G &= \frac{1}{D} [B V^{-1} \mathbf{1} - A V^{-1} m] \\ H &= \frac{1}{D} [C V^{-1} m - A V^{-1} \mathbf{1}], \end{aligned}$$

and $A = \mathbf{1}' V^{-1} m$, $B = m' V^{-1} m$, $C = \mathbf{1}' V^{-1} \mathbf{1}$, and $D = BC - A^2$.²

The expected return is by definition $w_p' m = \mu_p$ and its volatility is:

$$\sigma_p^2 = w_p' V w_p = \frac{1}{D} [C \mu_p^2 - 2A \mu_p + B]. \quad (6)$$

The portfolio that attains the minimum variance subject to constraint (2) but not (3) is the portfolio i with $\mu_i = A/C$ and $\sigma_i^2 = 1/C$.³ The mean-variance frontier is the part of the curve of Figure 1 where the expected return (Mean) satisfies $\mu_p \geq \mu_i$.

If risk were volatility, the minimum-variance portfolio problem is closely related to the optimization problem in which an agent maximizes expected utility with quadratic preferences (see Huang and Litzenberger, 1988; LeRoy and Werner, 2001).

Definition 2 *Portfolio \mathbf{q} is the optimal portfolio with quadratic preferences if its portfolio weight vector is the solution to the following constrained optimization:*

$$\max_w w' m - \frac{1}{2} \gamma w' V w \quad (7)$$

subject to (2).

²The optimal portfolio (5) admits short-sales (some of the weights may be negative). Short-sales can be seen as proxies for the use of derivatives by the portfolio manager.

³This portfolio is denoted by i in Figure 1.

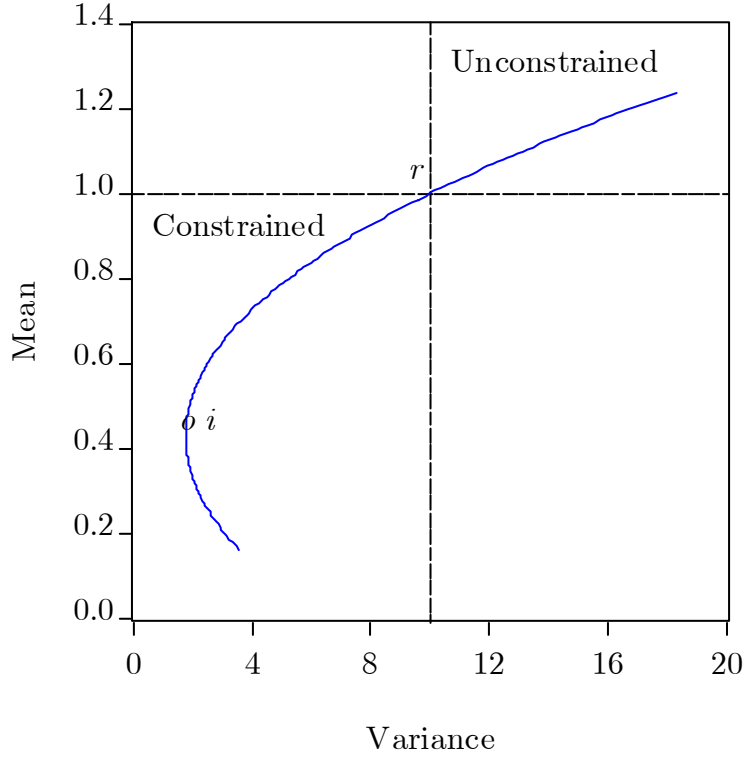


Figure 1: Mean-Variance Frontier: Constructed with the four instruments used by Bernstein and Chumacero (2006) that use monthly returns of fixed and variable income instruments of Chile and the US. An i denotes the minimum-variance portfolio. With maximum-expected return, maximum variance, or VaR constraint, the portfolios on the Unconstrained region can not be chosen and the constrained potfolio r is obtained.

The parameter $\gamma > 0$ defines the degree of risk aversion, with higher values indicating more of it. The solution to this problem is:

$$w_q = \frac{1}{\gamma} V^{-1} (m + E\iota), \quad (8)$$

where

$$E = \frac{\gamma - A}{C}.$$

Proposition 1 *Portfolio q belongs to the mean-variance frontier.*

Proof. Define $\mu_q = m'w_q$ and let γ be:

$$\gamma = \frac{D}{\mu_q C - A}. \quad (9)$$

Then, (8) can be expressed as:

$$w_q = G + H\mu_q,$$

which belongs to the mean-variance frontier. ■

Note that if μ_q were set equal to A/C (the minimum variance portfolio i), γ diverges in which case the problem is not well defined. Thus, $\mu_q > A/C$ must hold, which implies that portfolio i is not a portfolio q .

The impact of imposing other constraints to an agent that has quadratic preferences is considered next. Studying how portfolio selection changes when the manager faces other constraints is important as regulators may want to impose them as a response to potential agency problems. The most natural constraint would be an upper limit on the volatility of the portfolio. As it is shown, this constraint is equivalent to imposing an upper limit on the expected return. For expositional purposes, the proof of this equivalence starts by deriving this last portfolio.

Definition 3 *Portfolio \mathbf{r} is the optimal mean restricted portfolio with quadratic preferences if its portfolio weight vector is the solution to the following constrained optimization:*

$$\max_w w'm - \frac{1}{2}\gamma w'Vw$$

subject to (2) and

$$w'm \leq \underline{\mu}. \quad (10)$$

Proposition 2 *If $\underline{\mu} > A/C$, portfolio \mathbf{r} belongs to the mean-variance frontier.*

Proof. Using (9) note that if:

$$\underline{\mu} > \frac{D + \gamma A}{\gamma C},$$

constraint (10) is not binding and $w_r = w_q$. When this condition is violated, $\mu_q > \underline{\mu}$. In that case, $w_r = w_p$ for $\mu_p = \underline{\mu}$. ■

Definition 4 *Portfolio \mathbf{s} is the optimal variance restricted portfolio with quadratic preferences if its portfolio weight vector is the solution to the following constrained optimization:*

$$\max_w w'm - \frac{1}{2}\gamma w'Vw$$

subject to (2) and

$$w'Vw \leq \underline{\sigma}^2. \quad (11)$$

Proposition 3 *If $\underline{\sigma}^2 > 1/C$, portfolio \mathbf{s} belongs to the mean-variance frontier.*

Proof. Using (6) note that:

$$\underline{\mu} = \frac{A + \sqrt{D(C\underline{\sigma}^2 - 1)}}{C}$$

is the expected return consistent with $\underline{\sigma}^2$ in the mean-variance frontier. The proof follows from Proposition 2. ■

The previous propositions make clear that within this framework, imposing a constraint that precludes the volatility of a portfolio to exceed a threshold is equivalent to imposing a constraint on its expected return not to exceed a threshold. Figure 1 shows that these constraints imply that, within the quadratic preferences framework, the chosen portfolio would be either a portfolio on the mean-variance frontier in the constrained area (when the constraint is not binding) or it will be portfolio r (when the constraint is binding). Combinations of expected return and volatility on the unconstrained area would be unfeasible because they would violate the constraint.

2.2 Value-at-Risk and the mean-variance frontier

Value-at-Risk (VaR) has become a popular tool for risk management of financial institutions.⁴

Let $l(w)$ be the observed return of portfolio w . As the returns are random, so is $l(\cdot)$. Given the cumulative distribution of $l(\cdot)$, define the Value-at-Risk $[VaR(w, \alpha)]$ of portfolio w for a probability α as the value that produces:

$$\Pr[l(w) \leq VaR(w, \alpha)] = \alpha.$$

That is, the probability of obtaining a return of $VaR(w, \alpha)$ or lower is $\alpha\%$.⁵

If the returns follow an elliptic distribution with mean m and covariance matrix V ,⁶ then:

$$VaR(w, \alpha) = w'm_t + k_\alpha(w'V_t w)^{1/2},$$

with k_α being the quantile of level α of the distribution.⁷

⁴See Dowd (1998) for references.

⁵Oftentimes, $l(\cdot)$ is defined as a loss (instead of a return) and $VaR(w, \alpha)$ should be viewed accordingly.

⁶A multivariate elliptical distribution is fully characterized by its mean, covariance matrix, and characteristic generator. A linear combination of an elliptically distributed vector is also elliptical. Elliptical distributions are symmetric and unimodal, but are not constrained regarding kurtosis. Examples of elliptic distributions are the normal, Student-t, logistic, and Laplace distributions.

⁷If the returns follow a multivariate normal distribution $k_\alpha = z_\alpha = -z_{1-\alpha}$, where $z_\alpha = \Phi^{-1}(\alpha)$ (with $\Phi^{-1}(\cdot)$ denoting the inverse of the cdf of a standard normal distribution). For example, if $\alpha = 0.025$ and the returns are normal, $k_\alpha = -1.96$. If the returns follow a multivariate Student-t distribution with $v > 2$ degrees of freedom, $k_\alpha = z_\alpha(v)[(v-2)/v]^{1/2}$. In general, if the returns follow an elliptical distribution, VaR will be a linear function of the mean and standard deviation of the portfolio.

Definition 5 *Portfolio \mathbf{v} is the minimum VaR portfolio for a level α if its portfolio weight vector is the solution to the following constrained optimization:*

$$\max_w w' m_t + k_\alpha (w' V_t w)^{1/2} \quad (12)$$

subject to (2).

Proposition 4 *If $\alpha < 1/2$ and $k_\alpha < -\sqrt{D/C}$, portfolio \mathbf{v} belongs to the mean-variance frontier.*

Proof. Alexander and Baptista (2002, Proposition 1) show that if $k_\alpha < -\sqrt{D/C}$, portfolio \mathbf{v} exists and takes the form:

$$w_v = G + H\mu_v,$$

where:

$$\mu_v = \frac{A}{C} + \sqrt{\frac{D}{C} \left(\frac{(k_\alpha)^2}{C(k_\alpha)^2 - D} - \frac{1}{C} \right)}$$

which is in the mean-variance frontier. ■

As was the case with the \mathbf{q} portfolio, the minimum variance portfolio (i) is not VaR efficient given that $\mu_v > A/C$ must hold. Thus, if the distribution of the returns allows for the VaR function to be expressed as in (12), the \mathbf{v} portfolio can be expressed as a \mathbf{q} portfolio by setting:

$$\gamma = \left[\frac{1}{D} \left(\frac{C(k_\alpha)^2}{C(k_\alpha)^2 - D} - 1 \right) \right]^{-1/2}.$$

If VaR minimization were subject to the maximum volatility constraint (11), the resulting portfolio can be described as a \mathbf{v} portfolio resulting from an optimization with quadratic preferences and the same volatility constraint. If an additional VaR constraint of the form:

$$w' m_t + k_\alpha (w' V_t w)^{1/2} \geq \underline{VaR} \quad (13)$$

is considered, the resulting portfolio also belongs to the mean-variance frontier.

If this constraint is not binding, portfolio \mathbf{v} is selected; however, if is binding, there is a \underline{k} such that:

$$w' m_t + \underline{k} (w' V_t w)^{1/2} = \underline{VaR}.$$

If $\underline{k} < -\sqrt{D/C}$, the constrained portfolio will still be in the mean-variance portfolio and would be equivalent to an \mathbf{r} or \mathbf{v} portfolio, with a stricter volatility constraint that would be to the left of the constraint depicted in Figure 1.

3 Implications for VaR-based supervision

With elliptically distributed returns, VaR portfolio optimization or VaR limits are equivalent to maximum return or maximum volatility limits. This portfolio is equivalent to one obtained with a quadratic objective function. The resulting portfolio will be on the mean-variance frontier.

Next, we discuss some important implications of these results and consider potential effects for the practice of imposing VaR limits for pension funds.

- **Welfare:** In the case of pension funds, regulations over investment strategies such as VaR-based supervisions or quantitative limits, are often motivated as a response to a principal-agent problem. The regulator may consider that pension fund administrators (the agent) may be inclined to take riskier positions than what the affiliates (the principal) would prefer. The optimality of a particular regulation requires for the regulator to know the preferences of the principal and agent. With a competitive market and if the affiliates are free to choose their pension fund administrator, the less likely it is to observe differences on the incentives of the principal and the agents. Even with no principal-agent problems, regulation of investment strategies may be used because of the existence of moral hazard. For example, several governments provide minimum pension guarantees, which may induce both the principal and the agent to take riskier positions than in the absence of the guarantees. Thus, optimality and welfare considerations of a given regulation depend on how different are the preferences of the agent, the principal, and on the ability of the regulator to better approximate the preferences of the latter. Prior to imposing limits or similar regulations, the regulator should be clear with respect to the source of the problem and preferences and technologies of the agents involved .
- **Limits:** Bernstein and Chumacero (2006) demonstrate that quantitative limits are costly and inefficient mechanisms to limit the volatility of returns. This happens because quantitative restrictions imply a mean-variance frontier that is dominated by the mean-variance frontier without limits.

Figure 2 shows the mean-variance frontier of monthly returns in the case of Chile. The continuous line corresponds to the frontier with no limits, the dotted line forbids investing in foreign or domestic variable income instruments, and the dashed line forbids investing abroad.⁸ The distance between the lines will depend on how stringent are the limits. The minimum-variance portfolio of the restricted problem is to the left of the unrestricted one. Furthermore, the distance between the lines shortens on a given location depending on the specific limits imposed.

⁸At present, pension funds can not invest more than 30% of their portfolio abroad.

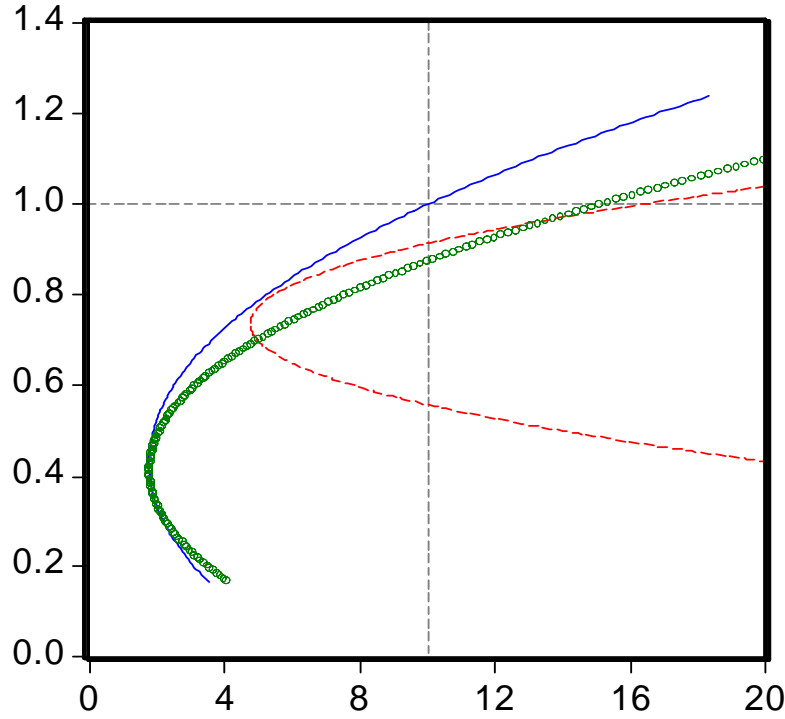


Figure 2: Effects of VaR and Quantitative Limits. The continuous line corresponds to the unconstrained mean-variance frontier. The dashed line corresponds to the mean-variance frontier with quantitative limits that prohibit investing abroad. The dotted line corresponds to the mean-variance frontier with quantitative limits that prohibit investing in variable income instruments.

Quantitative limits do not allow for proper diversification as, regardless of the risk aversion of the agents, limits lead to inefficient portfolios, as less volatility could be achieved with the same expected return in the absence of limits, or equivalently, obtain more return with the same volatility if no limits were imposed. For instance, more stringent limits imply lower risk allowed, at the expense of higher efficiency costs with respect to explicit volatility bounds. In some instances, quantitative restrictions might be justified. For example, limits on the investment on instruments issued by entities related to the pension fund manager or limits by issuer that prevent a pension fund from controlling a company may be desirable. Nevertheless, when imposing any restriction, costs and benefits should be carefully assessed.

If a VaR limit were imposed, the selected portfolio would be on the efficient frontier; however a stringent VaR limit may lead to a suboptimal allocation of

resources for agents that are less risk averse than the implied bound.

If both restrictions are imposed at the same time, the selected portfolio would be on the left side of the restricted frontier which implies an additional efficiency cost. In the example of Figure 2, an extra 1.3% annual return could have been obtained with the same volatility.

- **Ellipticity:** If the objective function is quadratic or the distribution of returns is elliptic, the portfolios chosen are on the mean-variance frontier. In those cases, a VaR limit will also be in the frontier (as long as the constraint is not too restrictive). Furthermore, with elliptically distributed returns Conditional VaR is equivalent to VaR (Rockafellar and Uryasev, 2000).

	FCh	VCh	FUS	VUS	Chile	US	All
Skewness	0.00	0.01	0.77	0.00	0.00	0.11	0.00
Kurtosis	0.00	0.21	0.00	0.00	0.00	0.10	0.00
Jarque-Bera	0.00	0.01	0.00	0.00	0.00	0.06	0.00

Table 1: P-values of Ellipticity Tests. FCh=Fixed Income (Chile). VCh=Variable Income (Chile). FUS=Fixed Income (US). VUS=Variable Income (US). Chile=Joint test for fixed and variable income instruments of Chile. US=Joint test for fixed and variable income instruments of US. All=Joint test for fixed and variable income instruments of Chile and the US.

If returns do not have a symmetric distribution, the mean-variance frontier may not be optimal (as long as preferences are not quadratic). Table 1 shows tests for skewness, excess kurtosis, and normality for each instrument with the Chilean example. As discussed, ellipticity does not preclude for excess kurtosis which is characteristic in financial time series. For the case of Chilean assets, normality is strongly rejected both for individual series and for its bivariate distribution. The same is (marginally) true for the US series. However, in the case of the US fixed income instrument, symmetry is not rejected.⁹ In the absence of symmetry, VaR portfolios are not in the mean-variance frontier. Furthermore, if agents take this characteristic into account, VaR may not be the best risk measure.

- **Dependence:** In practice, the VaR of a portfolio is computed with historical data. The historical VaR uses time series of returns expressed in the same (real) currency and term and assumes that the returns are independent. Efficiency would imply that this is not a bad assumption. However, using monthly data,

⁹Figure 3 presents additional evidence of the strong departures of normality of the series by comparing their empirical quantiles with the theoretical quantiles of the normal distribution. When normality is present, the dots should lie on the straight lines. The pattern of deviation from linearity provides an indication of the nature of the mismatch.

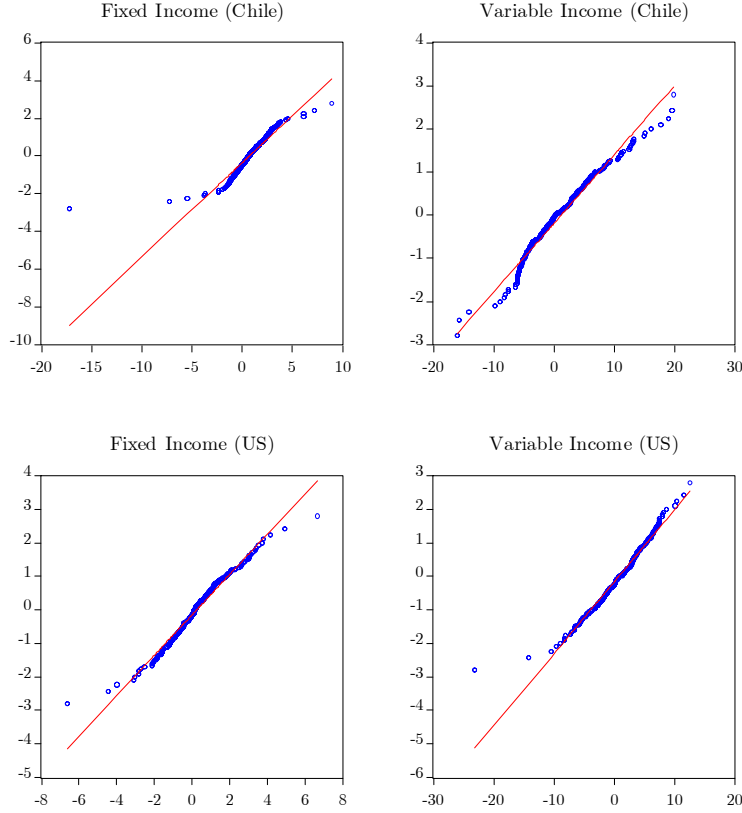


Figure 3: Theoretical (Normal) and Empirical Quantiles for Returns of Instruments

past returns help to forecast present returns. In that case, quantile estimates should consider this property. The same can be said with respect to second moments. ARCH/GARCH features are typical of financial returns. This implies that if VaR limits have the intention of limiting volatility, it should be consistently estimated using time series models. The statistical properties of the data are not properly taken into account with the historical VaR if dependence is present.

Figure 4 presents the residuals of GARCH(1,1) models for fixed and variable income instruments. These simple models present strong evidence of volatility clustering (calm and volatile periods tend to display persistence). Depending on the frequency and length of the observations used to compute the VaR measure, this kind of dependence may imply overly restrictive limits in high volatility periods and relatively loose limits on calm periods. As pension funds invest for the long run and therefore periods of short term high (low) volatility should not have a first order impact on the investment strategies of pension

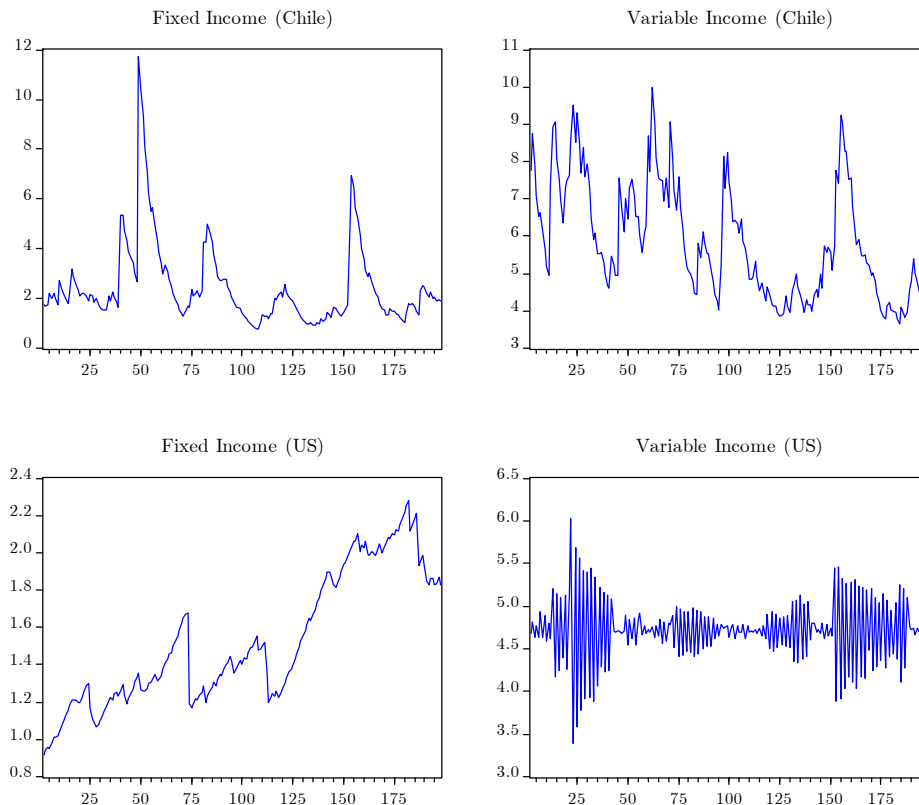


Figure 4: Projected Standard Deviation of Residuals in a GARCH(1,1) Model

fund managers.

- **Term:** Pension fund affiliates invest for their retirement and do not use the funds invested in the process. In this case, VaR limits should consider the term structure of risk. Guidolin and Timmermann (2006) demonstrate that the term structure of VaR varies according to the distribution of the returns.

For example, assume that the returns follow a multivariate normal distribution with mean vector m and covariance matrix V .¹⁰ In this case, no additional information regarding the distribution of the returns can be gathered with past data. If an agent decides to maintain the same portfolio for h periods, the mean and covariance matrix of the returns will be hm and hV respectively.¹¹

¹⁰As already discussed, time dependence in second moments and even in first moments make this an unrealistic assumption in practice. Furthermore, this example assumes that there is no risk free asset for the investing horizon and that fixed income instruments have a one-period maturity.

¹¹In a more general framework, the investor should compute the h periods ahead forecasts of the vector of expected returns and covariance matrix using the information available.

In the quadratic preferences set-up, the portfolio chosen would be the same regardless of the time horizon. This is because, the utility function is scaled by the factor h and the first order conditions do not depend on h . Thus, without changes in attitudes toward risk (γ) and under these restrictive assumptions, quadratic preferences imply that the portfolio that is optimal for a 1 period horizon is also optimal for any horizon. The standard deviation of a portfolio held for h periods would follow the square-root rule (as the standard deviation of that portfolio would be \sqrt{h} times the standard deviation of the portfolio held for one period).

On the other hand, the VaR of this portfolio becomes:

$$\begin{aligned} VaR(w, \alpha, h) &= hw'm + k_\alpha \sqrt{h}(w'Vw)^{1/2} \\ &= h\mu + \sigma k_\alpha \sqrt{h}. \end{aligned} \quad (14)$$

As the objective function is not proportional to h , the optimal portfolio will depend on h .

Take a portfolio, with expected return μ and volatility σ^2 . Differentiating (14) with respect to h , the horizon h^* where VaR attains a minimum is:

$$h^* = \frac{\sigma^2 (k_\alpha)^2}{4\mu^2}. \quad (15)$$

For example, if the returns are normally distributed and $\alpha = 0.025$, the value of h at which (14) attains a minimum is approximately equal to the square of the coefficient of variation of the portfolio. If the pair (μ, σ^2) is the tuple of expected return and volatility that would be optimally chosen for $h = 1$, (15) makes clear that that same portfolio can not be optimal for $h > 1$ as (14) will be increasing in h for $h > h^*$.

This implies that for $h > 1$, the first moment will tend to dominate the second. Thus, an investor maximizing (14) for $h > 1$ will choose more aggressive strategies (in line with the popular perception that the equity premium justifies more aggressive strategies for long term investors).

Figure 5 shows the importance of considering the investing horizon. The longer the investing horizon, the more aggressive the optimal \mathbf{v} portfolio will be. The equivalence between the \mathbf{v} and \mathbf{q} portfolio can be maintained by changing the value of γ in the objective function (7). As the second panel of the figure stresses, the risk aversion parameter γ should decrease with increases in h for the VaR objective function to be maximized.

An implication of this result is that VaR measures obtained from high frequency data for a relatively short span of time (say 1 or 2 years) when investors have different planning horizons may be dangerous. For long term investing horizons,

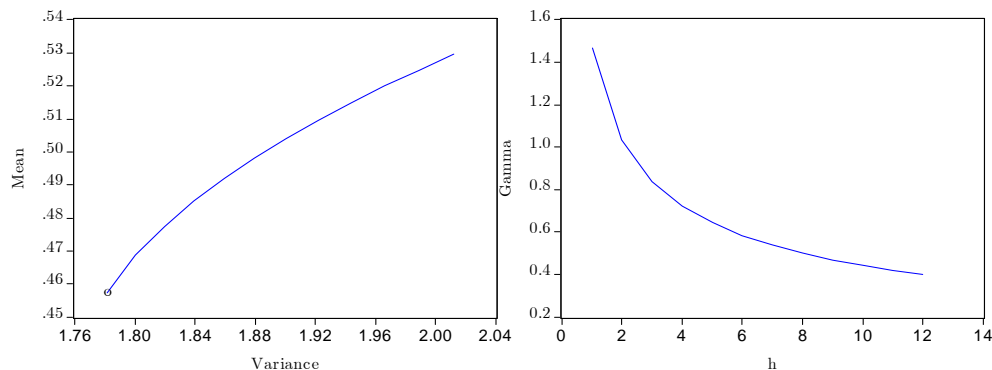


Figure 5: VaR and Term Structure. The left panel shows the combinations of (monthly) expected returns and volatility resulting of maximizing the VaR objective function for $\alpha = 0.025$ and $h = 1, \dots, 12$. The dot corresponds to the optimal portfolio for $h = 1$ that is consistent with quadratic preferences. The right panel shows the changes in γ that would be needed for the quadratic preferences portfolio to match the VaR portfolio when h changes.

it is needed to have consistent estimators of at least the unconditional first two moments of the distribution of returns. One of the requirements for this to happen is for the distribution to be ergodic for these moments and that the sample used covers a representative realization of “all states” of nature.

Additionally, as agents have different planning horizons, a universal VaR limit may be undesirable for some agents (particularly long term investors) as the second order considerations are not as important for them.

Multifunds (the ability of agents to choose from different portfolio strategies depending on the characteristics of the affiliates), with properly set varying VaR limits, may be an attractive alternative.

However, there is a final consideration that a regulator must consider when setting this type of limits for pension funds. The regulator may be interested on maximizing the pension attained with the accumulated resources. This embodies an annuitization risk at the moment of retirement. When retiring, a person usually buys an annuity. The price of the annuity at the moment of retirement depends, among others facts, on interest rates at that time. Therefore, the lower (higher) interest rates are at the moment of retirement, the higher (lower) the price of a unit of pension would be. Consequently, the same amount of accumulated funds would buy a lower (higher) pension. This is the same as saying that even when close to retirement, the investment horizon of a person is still significantly long. This should be taken into account at the moment of

setting restrictions to volatility.

4 Concluding remarks

This paper presents a framework for analyzing some of the implications that VaR-based regulation may have on how agents choose their portfolios.

It is shown that, under certain conditions, VaR limits can be seen as maximum expected return or maximum volatility constraints. In these cases, VaR portfolio strategies and VaR limits produce portfolios that are on the mean-variance frontier. The conditions under which these results hold are relatively restrictive and should be tested.

In terms of how VaR-based regulation is conducted for the case of pension funds, it is contended that more effort should be taken on quantifying the potential discrepancies between the principal (affiliate) and the agent (fund manager). This is crucial because, one of the main reasons for setting VaR limits would be that the agent is less risk averse than the principal. In principle, in such case, VaR limits in line with the preferences of the principal might be desirable. As risk aversion varies across systematic characteristics of the principal or planning horizons differ, a unique VaR limit is undesirable.

Furthermore, as this industry is characterized by long term investors, VaR limits may seriously affect pensions in the long run, because they not only restrict volatility, but also expected returns. Additionally, volatility or VaR limits might not be a good measure of the relevant risk phased by the future pensioner if the annuitization risk is ignored. A way to incorporate it, might be to express rates of return and volatilities in terms of units of pension; although this may be difficult to do in practice.

From a practical stand-point, regulators should make more efforts on obtaining precise estimators of the moments of the returns of assets, given that the availability of this information is crucial for setting an adequate VaR limit. Historical VaR computations using high frequency data for a short span period may not be relevant risk measures for most agents (considering their planning horizons).

When VaR limits are properly set, quantitative limits are undesirable. They preclude agents from diversifying their portfolios and conduce to suboptimal mean-variance combinations. However, as is the case with any regulation, costs and benefits should be assessed and restrictions relaxed when possible.

References

- Alexander, G. and A. Baptista (2002). “Economic Implications of Using a Mean-VaR Model for Portfolio Selection: A Comparison with Mean-Variance Analysis,” *Journal of Economic Dynamics & Control* 26, 1159-93.
- Berstein, S. and R. Chumacero (2006). “Quantifying the Costs of Investment Limits for Chilean Pension Funds,” *Fiscal Studies* 27, 99-123.
- Campbell, J., A. Lo, and C. MacKinlay (1997). *The Econometrics of Financial Markets*. Princeton University Press.
- Dowd, K. (1998). *Beyond Value at Risk: The New Science of Risk Management*. Wiley & Sons.
- Gourieroux, C., J. Laurent, and O. Scaillet (2000). “Sensitivity Analysis of Values at Risk,” *Journal of Empirical Finance* 7, 225-45.
- Guidolin, M. and A. Timmermann (2006). “Term Structure of Risk under Alternative Econometric Specifications,” *Journal of Econometrics* 131, 285-308.
- Huang, C. and R. Litzenberger (1988). *Foundations for Financial Economics*. Prentice Hall.
- LeRoy, S. and J. Werner (2001). *Principles of Financial Economics*. Cambridge University Press.
- Rockafellar, T. and S. Uryasev (2000). “Optimization of Conditional Value-at-Risk,” *Journal of Risk* 2, 21-41.